

1. A linear harmonic oscillator has mass 2 kg. It is driven by a force (in Newtons) of $8\cos\omega t$, where $\omega = 6 \text{ s}^{-1}$. The undamped resonant frequency is $\omega_0 = 5 \text{ s}^{-1}$. The damping is $\beta = 3 \text{ s}^{-1}$.

- What is the amplitude of the response?
- What is the phase shift?
- Plot the driving force and the response.

d. Now suppose the driving force is abruptly turned off at $t=0$. (It had been on since time immemorial.) Find the equation of motion for the mass after $t=0$. Sketch.

2. The oscillator in problem 1 is at rest. At time $t=0$, it is struck by a hammer, applying an impulse of $+6 \text{ kgm/s}$. You wish to apply a second impulse exactly $\pi/2$ seconds later to bring the oscillator to a halt. What magnitude and direction of impulse is necessary?

3. a) Find the Green's function for the critically damped oscillator.

b) Use your result in (a) to write down an integral that gives the position of a critically damped oscillator with $\omega_0 = 4 \text{ s}^{-1}$, mass = 4 kg, initially at rest, responding to a force (in N) given by

$$F = 8t, \quad 0 < t < 1 \text{ s.}$$

At $t = 1 \text{ s}$, what is the position and velocity of the mass?

5-2. If the field vector is independent of the radial distance within a sphere, find the function describing the density $\rho = \rho(r)$ of the sphere.

5-5. A particle falls to Earth starting from rest at a great height (many times Earth's radius). Neglect air resistance and show that the particle requires approximately $\frac{9}{11}$ of the total time of fall to traverse the first half of the distance.

5-7. Calculate the gravitational potential due to a thin rod of length l and mass M at a distance R from the center of the rod and in a direction perpendicular to the rod.

5-15. A particle is dropped into a hole drilled straight through the center of Earth. Neglecting rotational effects, show that the particle's motion is simple harmonic if you assume Earth has uniform density. Show that the period of the oscillation is about 84 min.

Show that it also works if a frictionless tunnel goes in a straight line from N.Y. - S.F. !